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The PSYCHOLOGICAL RECORD . . . .

JUNE, 1938
Vol. II No. 9

DIFFERENTIABLE MENTAL TRAITS

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BLOOMINGTON, INDIANA

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DIFFERENTIABLE MENTAL TRAITS*[†]

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During the past eight years, a series of experimental studies dealing with the identification and measurement of mental traits has been carried on in the Columbia laboratory. Most of these papers appeared before the expansion of Spearman's tetrad-difference method (by Thurstone, Hotelling and others) into the more general methods of factor analysis; and each stands more or less as a separate unit. Where factor methods were employed, their application was largely incidental to other techniques. For this reason, it has seemed desirable to bring together those experiments in which the size of the sample and the number and variety of the tests seemed to justify factor methods; and to apply one method of factor analysis consistently to all of our data. It was hoped that such an analysis would serve (1) to check previous findings, (2) to permit of generalization and summary, and (3) to indicate how much additional information may be expected, in general, from the factor as distinguished from other methods of analysis. It was possible to select studies in which children as well as adults were subjects. This enabled us to investigate the further problem of whether abilities tend to become more or less specific or independent as age increases.

In the present paper we have analyzed five studies: two dealing with verbal and number ability, two with memory, and one with verbal and number abilities as well as with memory. In one of the verbal-number studies children were subjects, and in the other adults were subjects; and the same division with respect to age obtained in the two memory studies.

I. STUDIES OF VERBAL AND NUMBER ABILITIES

1. Schneck's study (24).

For our purposes, the relevant data from this study may be summarized as follows:

(1) *Subjects.* There were 210 subjects, all male college students, ranging in age from 18 to 21 years.

* Prepared under the auspices of the Columbia University Council for Research in the Social Sciences.
† Manuscript accepted by Dr. J. G. Peatman, March 30, 1938.

(2) *Tests.* Nine tests were administered, five being verbal in character, and four numerical. The verbal tests were vocabulary, opposites, analogies, sentence completion, and disarranged sentences; the numerical tests were arithmetic reasoning, number series completion, equation relations, and mental multiplication. The entire battery required about five hours of testing time. Reliabilities were high, .90 or more for all of the tests except disarranged sentences, the reliability coefficient of which was .76.

(3) *Results.* By means of tetrad analysis applied to the verbal and number batteries separately, a verbal factor called V and a number factor called N were identified. Of the verbal tests the vocabulary test was most highly correlated with V, and of the number tests the arithmetic reasoning test was most highly correlated with N. The two factors were not quite independent: the correlation between scores in V and N, estimated by regression equations from the two batteries separately, was $.263 \pm .025$.

Table 1 gives the results of a factor analysis of Schneck's nine tests.* By means of Thurstone's centroid method (28), four factors have been calculated from the intercorrelations of the tests. The entries in the first four columns of Table 1 show the correlations of each test with the "centroid factors" I to IV. Considered geometrically, these entries are the projections of the nine test vectors upon the four arbitrary centroid axes (28, Chaps. I and III). As shown by the squares of the factor loadings in columns I^2 to IV^2, factor I accounts for 34% of the total variance of the battery; factor II for 15% of the variance, and factors III and IV for 2.9% and 2.4%, respectively. Taken together, the four factors account for about 54% of the total variance. The communalities of the nine tests (column h^2) indicate that from 90% (opposites) to 22% (disarranged sentences) of the variance of the individual tests is accounted for by the four centroid factors.

The contribution of factors III and IV to the total variance is negligible as compared with factors I and II (Table 1). It is probable that the factor loadings in III and IV represent minor bonds or group factors which cut across two or more of our tests. It would be interesting to identify these connections, but it is doubtful whether it could be done accurately enough with the present small battery to warrant the effort. Only two entries in column III^2 and only one in

* For intercorrelations, see Schneck, M. R. (24) Table 1, p. 22.

TABLE 1

FACTOR LOADINGS AND COMMUNALITIES OF SCHNECK'S NINE TESTS
(FOUR FACTORS, CENTROID METHOD)

Tests	I	II	III	IV	I^2	II^2	III^2	IV^2	h^2
1. Vocabulary	.776	.479	.174	.109	.602	.229	.030	.012	.873
2. Opposites	.776	.474	.247	.105	.602	.225	.061	.011	.899
3. Analogies	.677	.389	.092	—.107	.458	.151	.008	.011	.628
4. Sentence Completion	.679	.082	—.267	.218	.461	.007	.071	.048	.587
5. Disarranged Sentences	.325	.201	—.198	—.196	.106	.040	.039	.038	.223
6. Arithmetic Reasoning	.592	—.410	.032	.016	.350	.168	.001	.000	.519
7. Number Series Completion	.489	—.374	—.109	.260	.239	.140	.012	.068	.459
8. Equation Relations	.338	—.405	—.065	—.168	.114	.164	.004	.028	.310
9. Multiplication	.313	—.455	.183	.011	.098	.207	.033	.000	.338
Σk^2					3.030	1.331	.259	.216	4.836
$\frac{\Sigma k^2}{N}$					.337	.148	.029	.024	.537

column IV[2] contribute more than 5% to the communality of any test. The maximum contribution of either factor to a correlation coefficient is not greater than .066, most contributions being .01 or less. If we may safely disregard factors III and IV because of their small size, we shall be able to express the intercorrelations of our nine tests in terms of two common factors with considerable accuracy. Expressed geometrically, we may now describe our nine test vectors in terms of two major reference vectors, i.e., in a space of two dimensions.

Figure 1 represents an attempt to envisage the nine tests of our battery in terms of two "primary abilities." The diagram has been constructed by plotting the factor weights of each test in I and II as x and y coordinates with reference

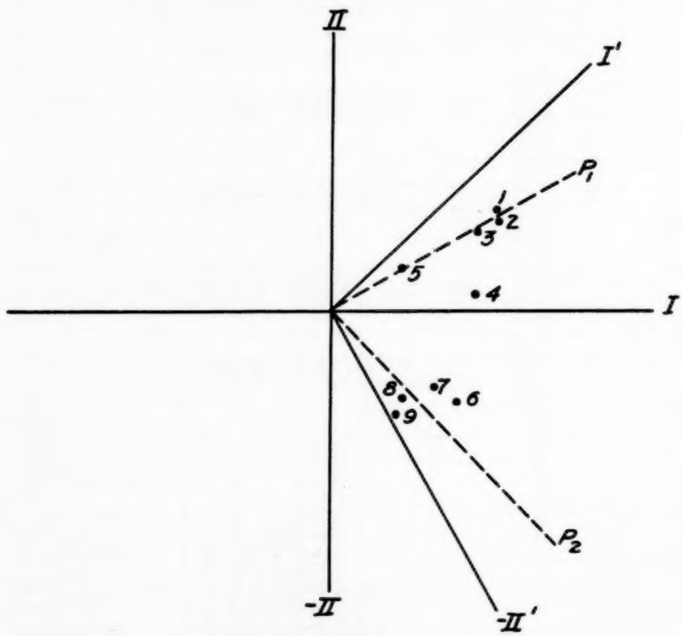

Figure 1

Plot of the Factor Loadings of the Nine Tests of Table 1 with reference to the Centroid Axes I and II.

to the centroid axes I and II. With the exception of test 4 (sentence completion) the verbal tests cluster in the first quadrant of Figure 1 and the number tests cluster in quadrant 4. The dotted line drawn through tests 1, 2, 3, and 5 locates primary ability P_1; and the dotted line through tests 6, 7, 8, and 9 locates primary ability P_2. The P_2 line has been drawn through the average of the x and y coordinates of tests 6, 7, 8, and 9. Tests 1, 2, 3, and 5 fall almost exactly upon line P_1, while tests 6, 7, 8, and 9 cluster around line P_2. The solid lines marked I′ and —II′ represent the new oblique reference vectors (normals) upon which the projections of the nine test vectors are measured. The matrix of the oblique transformation is

$$\begin{pmatrix} .688 & -.524 \\ .725 & .853 \end{pmatrix}^*$$

By means of this transformation the coordinates of our nine tests are determined with respect to the new axes, i.e., our test vectors are rotated into a new configuration.

The coordinates of the nine tests—their factor weights—with respect to the new reference axes are given in Table 2. Entries under I′ and II′ show the correlations of each test with the two hypothetical primary abilities. With the exception of test 4, the factor weights of the nine tests in Table 2 exhibit "simple structure" (28, Chap. VI), i.e., the test clusters reveal a definite pattern. Thus tests 1, 2, 3, and 5 have substantial weights in I′ (the "verbal factor") and zero weights in II′ (the "number factor"); while tests 6, 7, 8, and 9 have substantial loadings in II′ and zero weights in I′. The verbal tests contain only the one primary ability and the number tests contain only the other.**

From Table 2 it appears that except for test 4 a description of our battery in terms of two primary abilities is warranted. It is clear from Figure 1 that P_1 and P_2 are not strictly independent, though they are nearly so. The angle between these two vectors is 77 degrees. The cosine of an angle of 77° is .225; and this is the correlation between the two postulated primary abilities. The correlation would have

* Trait vector II has been reflected so that the projections of seven of the nine tests upon this reference axis have positive signs in Table 2.

** For an N = 210, the PE of an r of zero is .047. When N = 210, an r must be $\pm .141$ to differ by as much as ± 3PE from an r of .00. Hence, none of the weights of tests 6, 7, 8, and 9 in I′ (i.e., their projections upon I′) is significant, if test correlations with factors may be treated as ordinary product-moment r's.

TABLE 2

FACTOR WEIGHTS OF SCHNECK'S NINE TESTS IN THE TWO PRIMARY ABILITIES I′ AND II′

Tests	I′	II′
1. Vocabulary	.881	—.002
2. Opposites	.878	.003
3. Analogies	.748	.023
4. Sentence Completion	.526	.286
5. Disarranged Sentences	.370	—.001
6. Arithmetic Reasoning	.110	.660
7. Number Series Completion	.065	.575
8. Equation Relations	—.061	.522
9. Multiplication	—.115	.552
	(Verbal)	(Number)

been zero, of course, had the two abilities been strictly independent.

The reason why test 4 (sentence completion) behaves differently from the other verbal tests may perhaps be found in the discarded factors in Table 1. On both of the centroid axes represented by III and IV, sentence completion has the largest or next to the largest projection; while its projection upon axis II is virtually zero. The projection of this test upon axis III agrees in sign with disarranged sentences, and is opposed in sign to that of the opposites and vocabulary tests, both of which have reasonably large projections upon III. The projection of sentence completion upon axis IV agrees in size and in sign most nearly with the number series completion test. These clues suggest that sentence completion contains ingenuity (non-stereotype) elements not present in the other verbal tests; and that these constituents are shared with number series completion and with disarranged sentences. In a larger battery embracing a wider range of abilities, we should perhaps be able to isolate what here appear as minor group factors. Sentence completion's weight of .286 in II′ probably represents an unanalyzed reasoning or manipulative component which it shares with the number tests.

Several analyses besides the one here presented were made of the tests in Table 1. But the limited number of tests, and their lack of variety led us quickly to abandon the attempt to determine more than two factors. Oblique axes were used

in Figure 1 instead of orthogonal because they gave a cleaner break between the two hypothetical abilities. The results in Figure 1 and Table 2 not only uphold Schneck's finding of two distinct factors in his nine tests, but substantiate his further finding that these two abilities are not independent. Schneck's correlation of .263 between V and N (24, p. 36) compares favorably with our correlation of .225 between P_1 and P_2.

2. Schiller's study (23).

(1) *Subjects.* Both boys and girls were subjects in this experiment. In that part of the study considered here, the subjects were 189 boys in the third and fourth grades of a city public school. The average age was 9.15 years, the range in age being from 7 years 9 months, to 11 years 0 months.

(2) *Tests.* Twelve tests were administered, three of which may be classified as numerical, four as verbal, and five as non-language, spatial, and performance. The numerical tests were number series, arithmetic reasoning, and computation; and the verbal tests were vocabulary, analogies, sentence completion, and reading. The non-language tests consisted of adaptations of the Otis Primary and the Army Beta, the International Group Mental Test, and the Goodenough Drawing of a Man Test; and of four tests from the Pintner-Paterson series, viz., the five-figure, the casuist, Knox cube, and Healy A. Reliability coefficients ranged from .98 (computation) to .92 (analogies) for the numerical and verbal tests. Reliabilities were much lower for the non-language tests, averaging .70 for the Otis, Beta, International and Goodenough. The reliabilities of the four Pintner-Paterson tests were not determined, but were probably low.

(3) *Results.* By means of tetrad analysis, triad analysis, and a study of cross tetrads and mean tetrads, three factors, V (verbal), N (numerical), and S (spatial) were regarded as established. Evidence for the differentiation of S from V and from N was much more conclusive than evidence for the differentiation of V from N (23, pp. 53-55).

Table 3 gives the factor loadings of Schiller's tests in each of the first four centroid factors. Taken together, these four factors account for 65% of the variance of the battery. To this total, III and IV contribute less than 5%. Factor III is heaviest, on the average, in the numerical tests; but both III and IV come close to being specifics. Test 1 (number series) is the only test having a loading in III greater than 5%; and test 7 (reading) is the only test having a loading

TABLE 3

FACTOR LOADINGS AND COMMUNALITIES OF SCHILLER'S TWELVE TESTS
(FOUR FACTORS, CENTROID METHOD)

Tests	I	II	III	IV	I^2	II^2	III^2	IV^2	h^2
1. Number Series	.750	—.038	.281	—.233	.563	.001	.079	.054	.697
2. Arithmetic Reasoning	.827	—.309	.111	—.119	.684	.095	.012	.014	.805
3. Computation	.731	—.143	.222	.084	.534	.020	.049	.007	.610
4. Vocabulary	.766	—.353	—.203	.088	.587	.125	.041	.008	.761
5. Analogies	.810	—.238	—.054	—.023	.656	.057	.003	.001	.717
6. Sentence Completion	.839	—.314	—.098	—.014	.704	.099	.010	.000	.813
7. Reading	.743	—.341	—.175	.301	.552	.116	.031	.091	.790
8. Otis	.679	.355	.035	.067	.461	.126	.001	.004	.592
9. Beta	.636	.466	.144	.110	.404	.217	.021	.012	.654
10. International	.768	.163	.133	.153	.590	.027	.018	.023	.658
11. Goodenough	.473	.298	—.157	—.153	.224	.089	.025	.023	.361
12. Performance	.425	.387	—.083	—.154	.181	.150	.007	.024	.362
Σk^2					6.140	1.122	.297	.261	7.820
$\frac{\Sigma k^2}{N}$					.512	.094	.025	.022	.652

in IV greater than 5%. It is probable that III and IV are too small to have definite meaning, and should be discarded. However, III was retained because, as will be shown later, it was possible to make psychological sense out of it. Rules setting up criteria to be followed in deciding how many factors may profitably be calculated from one's data have been laid down (14, p. 494) but their application is necessarily arbitrary. How many factors to compute will depend upon the size and reliability of the correlations, the number and variety of the tests, and the likelihood of interpreting additional factors psychologically. The range of functions sampled by the tests in Table 3 enabled us to interpret factor III, while in Table 1, a third factor of about the same numerical size could not be interpreted and hence was discarded.

All twelve of our tests have positive weights in I. This is to be expected, of course, since the projections of the test vectors upon the first centroid axis are directly dependent upon the intercorrelations of the tests themselves, all of which are positive.* The signs of the factor loadings in II are of interest. The projections of all of the verbal and numerical tests upon axis II are negative, while the projections of the five non-language tests upon II are positive. This suggests a break or differentiation between the verbal-number tests on the one hand, and the non-language and performance tests on the other. This fact, and the small values in columns III and IV, made it seem probable that our twelve test vectors could be rotated into a meaningful configuration in two dimensions. But this did not prove to be feasible. Several transformations, orthogonal as well as oblique, were carried out; but in no case were we able to get a configuration which exhibited definite structure.

An attempt was made, next, to represent the intercorrelations of our twelve tests by three common factors, i.e., to describe the twelve test vectors in a space of three dimensions. There appeared to be considerable justification for retaining axis III instead of IV besides the fact that its factor loadings are slightly larger. First, the projections of the verbal and number tests upon III are separated as to sign; secondly, the projections of the number tests upon III, though small, are larger than the projections of the verbal

* The factor weight of any test in I is the sum of its *r's* with the other tests divided by the square root of the sum of the *r's* in the table (26, p. 3).

and spatial tests upon the same axis. Finally, the configuration obtained by plotting the entries in II against the entries in III gave the nearest approach to simple structure. Figure 2 presents the situation graphically. In this diagram, which represents the cross section of a three-dimensional sphere, the augmented factor loadings (28, pp. 118, 167) of II have been plotted against the augmented factor loadings of III. The first centroid axis (I) is represented by a line perpendicular to the plane of II and III. To one looking down upon the sphere from above, the tests would appear as dots upon the surface. Inspection of the arrangement of these dots reveals that, in general, they fall into three fairly discrete groups: points 1, 2, and 3 (number tests) lie in quadrant 2; points 4, 5, 6, and 7 (verbal tests) in quadrant 3; and points 8, 9, 10, 11, and 12 (non-language tests) group around

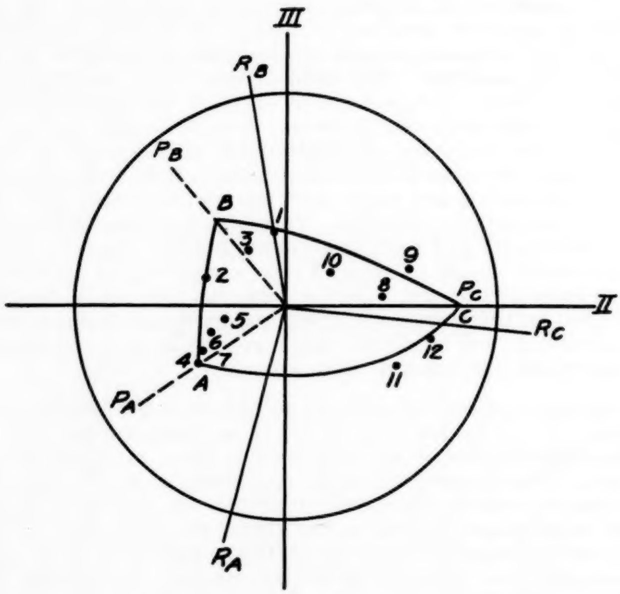

Figure 2

Plot of the Factor Loadings of the Twelve Tests of Table 3, with reference to the Centroid Axes II and III. (Axis I is perpendicular to the plane II-III.)

axis II in quadrants 1 and 4. If three great circles AB, AC, and BC, are drawn upon the surface of the sphere, as represented in Figure 2, each test vector will lie in, or at least close to, one of the three planes determined by these circles.

The coordinates of the three primary ability vectors, P_a, P_b and P_c are given in Table 4.

TABLE 4

	P_a	P_b	P_c
I	.889	.865	.648
II	—.397	—.348	.762
III	—.228	.360	.000

Each column in Table 4 represents the coordinates of one primary vector. The projections of these three primary vectors upon the plane of the centroid axes II and III are shown in Figure 2.

The coordinates of the three new reference vectors (axes) are given in Table 5.

TABLE 5

	R_a	R_b	R_c
I	.292	.182	.395
II	—.248	—.155	.917
III	—.943	.978	—.060

The projections of these three reference axes upon the plane of the centroid axes II and III are represented in Figure 2. R_a is perpendicular to the plane BC; R_b is perpendicular to the plane AC; and R_c is perpendicular to the plane AB. The entries in Table 5 are the direction cosines of these three planes, i.e., the direction cosines of the normals (here the reference axes) to these planes (28, pp. 33-38). The coordinates of the three reference vectors (Table 5) were determined by combining the "graphical method" (28, pp. 167-170) and the "method of averages" (28, pp. 175-178) with a certain amount of experimental fitting. An effort was made to keep all of the points within the spherical triangle shown in Figure 2, in order that the new factor weights (i.e., the projections of the test vectors upon the new axes) might all be positive.

TABLE 6

FACTOR WEIGHTS OF SCHILLER'S TWELVE TESTS IN THE THREE PRIMARY ABILITIES A, B, C

	Tests	I′ A	II′ B	III′ C
1.	Number Series	—.037	.418	.244
2.	Arithmetic Reasoning	.213	.308	.036
3.	Computation	.040	.372	.144
4.	Vocabulary	.503	—.004	—.034
5.	Analogies	.346	.132	.105
6.	Sentence Completion	.415	.106	.109
7.	Reading	.467	.017	—.009
8.	Otis	.077	.103	.592
9.	Beta	—.076	.185	.669
10.	International	.059	.245	.444
11.	Goodenough	.212	—.114	.469
12.	Performance	.106	.064	.528
		(Verbal)	(Number)	(Spatial-Performance)

Table 6 gives the coordinates of our twelve tests (their factor loadings) with respect to the new reference axes R_a, R_b, and R_c. The entries in columns I′, II′, and III′ give also the correlations of each test with the three hypothetical primary abilities, verbal, number, and spatial-performance. The test vectors represented by the factor loadings in Table 6 exhibit simple structure (28, Chap. 6) with the possible exception of tests 5, 6, and 11, and it is highly probable that these tests also conform.* Each test has a zero or nearly zero loading in at least one column, and several tests, e.g., 4 and 7, have zeros in two columns. The structure revealed in Figure 2 and in Table 6 is evident enough but is not unique. Plane AC is not uniquely determined as only two tests lie along the arc AC; and B and C are located in part by the test vectors and in part by the reasonableness of the configuration which they give. Furthermore, as pointed out above, the centroid factor weights in III are quite small and unreliable; hence any analysis based upon them is necessarily subject to question.

* For N = 189, the PE of an r of .00 is .049. Hence, an r must be greater than $\pm$.147 in order to represent a deviation of as much as $\pm$3PE from an r of .00. (See footnote, p. 263).

In justification of the analysis in Table 6, however, it may be offered that the factor weights shown in Table 6 jibe with other evidence. The new reference axis I′ has been taken to represent the verbal factor; axis II′ the number factor; and axis III′ the spatial, non-language and performance factor. Note that the four verbal tests, 4, 5, 6, and 7 have strong factor weights in I′; while test 2 (arithmetic reasoning) and test 11 (Drawing a Man) have moderate weights in I′. The reading and comprehension required for solving problems in arithmetic (especially for young children) are sufficient to account for the loading of test 2 in I′, but the "verbality" of test 11 seems anomalous unless facility with verbal concepts aids performance in drawing a man. That this is actually true seems probable from Goodenough's analysis (13) of the mental processes operative in children's drawings. Goodenough reports also a correlation of .73 (13, p. 50) between Stanford-Binet MA and drawing test MA for 37 nine year olds; and an r of .75 between the same two variables for 334 children ranging in age from 4 to 10 years. Evidently, then, verbal intelligence as measured by Stanford-Binet, and drawing ability as measured by the Goodenough test are highly related in children.

The three number tests are most heavily weighted in II′,—the number factor. Test 5 (analogies) and test 6 (sentence completion) have small and probably unreliable loadings in number ability (see also p. 264). The small weights of Otis and Beta and the fairly large weight of International in the number factor seem not unreasonable in view of the symbol manipulation demanded by these tests. The five non-language tests have their heaviest loadings in III′ (the spatial-performance factor). Test 3 (computation), test 5 (analogies) and test 6 (sentence completion) also have small, and probably unreliable, weights in III′. Number series completion is reliably represented in III′, due perhaps to the logical arrangement and relation finding demanded by this test.

As has been pointed out above, the three reference axes R_a, R_b, and R_c (Figure 2) are normals to the three planes defined by the great circles whose arcs are pictured in Figure 2 as AB, AC, and BC. While these reference vectors may be orthogonal, their orthogonality is highly improbable and should be tested if we wish to know whether our primary abilities P_a, P_b, P_c, are uncorrelated. The cross products of the entries in the columns of Table 4 give, respectively, the cosines of the angles between vectors P_a and P_b; P_a and

P_c; and P_b and P_c. And the cosines of the angles between these vectors give the correlation between the abilities represented by P_a, P_b, and P_c (27). As one might surmise from Figure 2, P_a and P_b are most highly correlated. The angle between these two radii in our three-dimensional sphere is 34°, the cosine of which is .825. The correlation between the hypothetical verbal and numerical abilities in our population of 9 year olds is, therefore, .825; and this close relationship is substantiated by reference to Schiller's original data. This author reports (23, p. 32) an average intercorrelation between her three numerical and four verbal tests of .629. Since the four verbal tests had an average intercorrelation of .735, and the three numerical tests an average intercorrelation of .641, the estimated correlation between what is common to the verbal and number batteries is $\frac{.629}{\sqrt{.735 \times .641}}$ or .917. This correlation of .917 is not directly comparable to the correlation of .825 between our two primary abilities because it includes minor common factors which do not enter into the latter correlation. Moreover, the correlation of .917 is probably too high as an estimate of communality in the two sets of tests in any case, as we have used average correlations of different test groups as reliability coefficients. The correlation of .825 is not, then, an unreasonable estimate of the communality of our verbal and number abilities.

The correlation between P_a and P_c—verbal and non-language abilities—is .273; and the correlation between P_b and P_c—numerical and non-language abilities—is .296. These *r's* may be compared with the average intercorrelations of .330 found by Schiller between her four verbal and five non-language-performance tests; and the average intercorrelation of .414 between the three number and five non-language tests. As mentioned above, these latter *r's* are probably somewhat inflated as a result of the method employed in deriving them.

Our analysis of Schiller's tests agrees in general with her finding;—namely, that her twelve tests contain three factors, verbal, numerical, and non-language-spatial. Factor analysis, when oblique reference axes are used, indicates more clearly than did tetrad analysis the lack of independence among these three primary abilities, and their degree of overlap. While there is no "general intelligence" in the sense of a single primary ability, the verbal and numerical tests taken together constitute, in effect, a highly

integrated general factor which is relatively independent of the performance and non-language tests (22).

The high correlation (.825) between the verbal and number factors when children are subjects presents an interesting contrast to the low correlation (.225) between the same two factors (see p. 265) when adults are subjects. This drop in correlation with age may be attributed (1) to differences as between the two groups; or (2) to a true tendency toward independence of these two factors as age increases. In favor of the first hypothesis, it may be said that Schneck's sample was undoubtedly highly selected and restricted in range as compared with Schiller's Fourth Grade children; that the tests measuring the verbal and number factors in the two groups while comparable in form were quite different in content and difficulty; and that we have no way of knowing what the r between the verbal and number factors in Schiller's group will be when these children are 21 years old. These objections offer real—but not insurmountable—difficulties to the notion of increasing independence of verbal and number abilities with increase in age. In favor of the latter thesis, we may cite the recent study of Asch (3b). Asch compared the correlations between various tests of verbal and number abilities administered to 79 boys, first, when the subjects were 9 years old, and again when they were 12 years old. The average drop in correlation for the eight tests over the three year period was .15 (from .56 to .41); the drop in the correlation between vocabulary and arithmetic for the same period was .20 (from .66 to .46); and between number series completion and vocabulary .17 (.42 to .25). In general, the reduction in correlation between the verbal and number tests was greater than the reduction in correlation within either the verbal or number tests taken as separate groups. Garrett, Bryan, and Perl (11b) have also found progressive reduction in the intercorrelations of a variety of mental tests administered to groups of nine, twelve, and fifteen year old children. The drop in correlation between vocabulary and arithmetic, from nine to fifteen, was .15 (.52 to .37) for groups of approximately 100 boys (108 at 9, 102 at 15). While these comparisons are not made upon the same groups, as in Asch's study, there was no significant reduction in the variability of either the vocabulary or the arithmetic test with age increase which might account for the loss in correlation. Except for age, the groups were well matched for general intelligence level and for socio-economic status.

From the experimental data cited above it seems probable that the reduction (from .825 to .225) in the correlation between the verbal and number factors observed above represents at least in part a true tendency toward independence of these primary abilities as age increases. How much of the drop in correlation is to be attributed to the increasing "independence" of our variables and how much to group differences it is impossible to say. But it appears likely that not all of the reduction is due to the latter factor.

II. STUDIES OF MEMORY

1. Anastasi's first study (2).

(1) *Subjects.* Two hundred and twenty-five male college students were subjects in this study. The age range was from 17 to 28 years with the mean close to 20 years.

(2) *Tests.* Eleven tests in all were administered, eight tests of rote or associative memory, and three tests intended to measure abilities relatively independent of associative memory. The memory tests consisted of four paired associate tests, word-word, picture-number, form-number, and color-word; and of four other memory tests, digit span, retained members (four letter words), recognition of forms (geometrical), and recognition of nonsense syllables. The "non-memory" tests were vocabulary, arithmetic reasoning, and the Minnesota Paper Form Board. The reliabilities of the color-word, retained members, digit span, form recognition, and syllable recognition were low, .631, .728, .745, .539, and .675, respectively. The other three memory tests had reliabilities above .80 and the vocabulary, arithmetic, and form board tests reliabilities above .90.

(3) *Results.* From a tetrad analysis of six of her eight memory tests, Anastasi revealed the presence of overlapping group factors (2, p. 44), which cut across two or more of her tests. In order to isolate the central memory factor, an attempt was made to render these group factors specific by combining overlapping tests. The following combinations were made:

1. Word-word
2. Picture-number—form-number
3. Color-word—retained members (words)
4. Syllable recognition.

The tetrads yielded by these four test groups, while large in two instances (2, p. 47), were not quite statistically significant; and hence were taken to be indicative of a common

memory factor. The correlations of the test-groups with the central memory factor were as follows: r_{1m} = .662; r_{2m} = .772; r_{3m} = .736; r_{4m} = .456. The multiple R of the battery with memory was .894. That the central memory factor is not entirely "verbal memory" is shown by the fact that the second and third groups of tests had the highest correlations with "M."

Table 7 gives the weights of each of the eleven tests in the first four centroid factors calculated from the *corrected* intercorrelations of Anastasi's tests (2, p. 39). A fifth factor was small and is not shown. Factor analyses were made of obtained as well as of corrected correlations, but only the results for corrected *r's* are shown in Table 7. Corrected *r's* were used because of the low reliability coefficients of the memory tests. Furthermore, it was hoped that by removing chance errors the very small communality of test 6 (digit-span) and of test 9 (vocabulary) might be increased. This last hope was only partly realized. To be sure, the communality of test 9 was raised slightly (from .060 to .079), but the communality of test 6 (digit-span) was actually lowered (.169 to .144). Lack of relationship with the other memory tests is shown consistently by digit-span. The average intercorrelation of this test with the other tests was .107 (obtained) and .143 (corrected). Anastasi comments upon this test as follows (2, p. 40): "......it seems probable that digit-span is not primarily a test of memory for an adult group of subjects, performance on it probably depending on incidental factors, such as attention, an active attitude, desire to do well on the test, etc." The low communality of the vocabulary test was not unexpected in view of the decided non-verbality of part of the memory battery and of the other two reference tests.

Examination of Table 7 reveals the difficulty which one encounters in attempting to analyze a battery of this sort in terms of common factors. Disregarding for the moment tests 6 and 9 because of their low communality, nine tests are left of which seven were designed specifically to measure associative memory. If tests of other and very diverse capacities had been included in the battery, the common memory factor suggested by the like-signed factor weights in III would undoubtedly stand out more clearly. As it is, a common memory factor (if present) is probably obscured by minor group factors which cannot be isolated by the present battery. In spite of these complications, a memory factor is apparently indicated by the opposition in signs in

TABLE 7

FACTOR LOADINGS IN THE FIRST FOUR CENTROID FACTORS, AND COMMUNALITIES, OF ANASTASI'S ELEVEN TESTS (CORRECTED CORRELATIONS)

Tests	I	II	III	IV	I^2	II^2	III^2	IV^2	h^2
1. Word-word	.686	.260	—.259	.115	.470	.068	.067	.013	.618
2. Picture-number	.680	—.169	—.379	.297	.462	.029	.144	.088	.723
3. Form-number	.767	—.123	—.105	.254	.588	.015	.011	.065	.679
4. Color-word	.773	.193	—.302	—.218	.598	.037	.091	.048	.774
5. Retained members	.762	.323	—.146	—.271	.581	.104	.021	.073	.779
6. Digit-span	.260	.107	.248	.053	.068	.011	.062	.003	.144
7. Form recognition	.633	—.358	—.227	—.229	.401	.128	.052	.052	.633
8. Syllable recognition	.578	.247	.221	.210	.334	.061	.049	.044	.488
9. Vocabulary	.134	.128	.172	.122	.018	.016	.030	.015	.079
10. Arithmetic	.333	—.235	.363	.114	.111	.055	.132	.013	.311
11. Minnesota	.350	—.375	.301	—.292	.123	.141	.091	.085	.440
Σk^2					3.754	.665	.750	.499	5.668
$\frac{\Sigma k^2}{N}$					.341	.060	.068	.045	.515

III, and a language vs. non-language factor by sign opposition in II. The factor loadings in IV are too large to be ignored (five are greater than .05), but it is impossible from Table 7 to tell precisely what function they represent.

No clear-cut structure was revealed by plotting II vs. III, II vs. IV, or III vs. IV,—all around I as a pivot. Since no definite clusters of tests were immediately apparent from these diagrams, a series of rotations were made in accordance with what appeared to be logical considerations. First, the factor weights of the eleven tests in I were plotted against the factor loadings in III, after trait vector III had been reflected so as to give plus signs to the loadings of the memory tests. Tests 1, 2, 3, 4, 5, and 7 fell in the first quadrant of our diagram, and tests 6, 8, 9, 10, and 11 fell in quadrant 4. A counter clockwise rotation of approximately 37° around axes II and IV brought all of the tests into a positive region with tests 10 and 11 (arithmetic reasoning and Form Board) falling close to axis —III′; and test 2 close to axis I′. The matrix of the orthogonal transformation was

$$\begin{pmatrix} .800 & -.600 \\ .600 & .800 \end{pmatrix}^{*}$$

The loadings of the eleven tests in I′ (rotated I) appear in Table 8 in the first column. (Since we can number our rotated axes as we like, this axis might be marked II′, III′, or IV′. It is called I′ here because it carries the memory loadings.) The factor loadings in III′ which seem to embrace a conglomerate facility with words and numbers as well as spatial forms, were next plotted against II (which apparently involved words as opposed to forms) in order if possible to separate out the language factor. In this diagram another orthogonal transformation seemed called for, since tests 1, 4, and 5 fell close to the vector separated by 90° from the vector through test 11. The transformation matrix was

$$\begin{pmatrix} .640 & -.769 \\ .769 & .640 \end{pmatrix}^{**}$$

The new second factor weights of the eleven tests appear in Table 8 under II′,—which is designated the word or language factor. This factor is heaviest in the word-word, retained members (words), and syllable recognition tests, and

* Trait vector III′ was reflected in order to change the signs of its factor loadings in Table 8.

** Trait vector II′ was reflected in order to change the signs of its factor loadings in Table 8.

lightest in the picture-number, form-number, form recognition, arithmetic and paper form board tests. It seems clearly to be identified with word manipulation or word facility.

The factor loadings of the just rotated vector II were plotted against the factor weights in centroid vector IV, in order, this time, to separate the form from the number factor. It was impossible to find a rotation which would make all of our factor weights positive, but a counter clockwise rotation of 57° made all of the tests but 1, 4, and 5 positive. The orthogonal transformation matrix was

$$\begin{pmatrix} .537 & -.844 \\ .844 & .537 \end{pmatrix}^{*}$$

The new third and fourth factor loadings appear in Table 8 under III′ and IV′. It appears that III′ is concerned primarily with number manipulation, and IV′ with spatial or form relations, but these designations are not very clear cut.

Various transformations, orthogonal and oblique, were tried with the four centroid axes in Table 7 in an effort to discover structure in our variables. The orthogonal transformations, the results of which are shown in Table 8, seemed to be the most logical, the most obvious, and the best calculated to make psychological sense. Thurstone (28, pp. 155-156) has laid down three conditions for unique simple structure. When there are four trait vectors (as in the present instance) these requirements are that (1) each row of the table must contain at least *one* zero; (2) each column must contain four or more zeros; and (3) for every pair of columns there must be at least four tests which have zero weights in one column, but not in the others. Examination of Table 8 reveals that these conditions are fairly well met. Furthermore, and more important psychologically, the zero entries fall where we should logically expect them to occur. Thus, those memory tests (except 8) which deal predominantly with words, i.e., 1, 4, 5, have non-significant or zero entries in III′ (number) and IV′ (form). In contrast, the loadings of these tests in trait vector II′ (words) are all significant. Those memory tests which deal primarily with numbers, i.e., 2, 3, and 6, have significant weights in III′ (number) and non-significant weights in I′ and IV′. The form recognition test has its heaviest weight

* Trait vector IV′ was reflected in order to change the signs of its factor loadings.

TABLE 8

NEW FACTOR WEIGHTS OF ANASTASI'S ELEVEN TESTS (TABLE 7) AFTER A SERIES OF ORTHOGONAL TRANSFORMATIONS

Tests	I′	II′	III′	IV′
1. Word-word	.703	.331	.093	—.069
2. Picture-number	.771	—.063	.352	.001
3. Form-number	.677	.146	.412	.175
4. Color-word	.799	.290	—.159	.157
5. Retained members (words)	.698	.466	—.200	.192
6. Digit-span	.059	.309	.155	.144
7. Form recognition	.642	—.148	.012	.445
8. Syllable recognition	.329	.525	.309	.094
9. Vocabulary	.004	.238	.149	.007
10. Arithmetic reasoning	.048	.133	.379	.384
11. Minnesota Paper Form Board	.099	.001	.069	.652
	(memory)	(word)	(number)	(form)

in IV′ (form), while test 3 (form-number) has an almost significant loading in IV′. There are five negative loadings in Table 8, of which only one is definitely significant. When N = 225, the PE of an r of .00 is .045. Hence, an r must be greater than ±.135 (±3PE) to be probably significant; and greater than ±.180 (±4PE) to be certainly significant. Of the negative weights in Table 8, three (those of tests 4, 5, and 7) are probably significant. These negative loadings are troublesome, but not unreasonable.

We have designated the second, third, and fourth columns in Table 8 "word," "number" and "form" factors, respectively. This labeling of factors in terms of the material involved in the tests in which they appear seems justified by the work of Smith (25) and Hertzman (16). These authors found that material (e.g., words, numbers, spatial forms) was more important in making for test clusters than the operation of what seemed analytically at least to be common mental processes.

The three "non-memory" tests in Table 8 fit quite well into the structure which we have built up. The Minnesota Paper Form Board exhibits a small memory factor and a relatively large "form" factor. The arithmetic reasoning test has fairly large weights in both form and number, and probably a non-significant weight in the "word" factor;—the latter due in part, perhaps, to the reading element in the arithmetic test. The vocabulary test, as already pointed out, has so small a degree of communality with the other tests that all of its weights are open to question. However, it is interesting to note that vocabulary's largest weight is

in the "word" factor (as it should be); and that its weights in "number" and "form" are probably non-significant. All three of the non-memory tests have negligible weights in the memory factor. This result is in agreement with Anastasi's finding (2, p. 42) that the mean correlation of vocabulary with her memory battery was .058; of arithmetic with the memory battery, .147; and of the Minnesota Form Board test with the memory battery, .145. All of these correlations had been corrected for attenuation and hence represent maximum values. Perhaps the low correlations of these tests with memory is attributable in part to the character of the sample, and in part to the character of the tests. Anastasi's subjects were all college students (superior adults) for whom we may fairly assume a common knowledge of the actual materials appearing in the vocabulary, arithmetic, and paper form board tests. It is improbable that recall memory would play a very large role in these tests; and this is apparently what actually happened.

In the process of getting a measure of the central factor, Anastasi found that her tetrads indicated the presence of fairly strong group factors among her tests. These bonds she attempted to eliminate or render specific by combining those tests which seemed to possess common group factors; but even after making various combinations, her tetrads were still uncomfortably large (2, p. 47). If Table 8 is even a reasonably accurate picture of the constitution of Anastasi's memory battery, it is readily perceived why this was true. In addition to the common memory factor I′, for instance, the test combination 2 and 3 (page 274) overlapped test 8 in III′ and test combination 4 and 5 in IV′. Also, test combination 4 and 5 contained II′ which was also shared by tests 1 and 8. In short, Anastasi's central memory factor probably embraced more than associative memory. Further evidence of this possibility is found in the fact that Anastasi's correlations between her test combinations and her central factor are slightly higher than the correlations between her tests and our primary ability I′, although the latter *r's* have been corrected for attenuation. In this study, factor analysis supplies considerably more information than did tetrad analysis.

2. Anastasi's second study (3).

(1) *Subjects.* In this study the subjects were 140 women college students, 92% being sophomores, the remaining 8% juniors and freshmen. The age range was from 16 to 28 years, the mean age being 18.6 years.

(2) *Tests.* Four tests of memory, two tests of verbal and two tests of number abilities, were administered. The memory tests were the word-word, picture-number, syllable recognition, and retained members (words). These were the four single tests which most nearly satisfied the tetrad criterion for a general factor in Anastasi's previous study (2, p. 47). The two verbal tests (vocabulary and analogies) and the two number tests (number series completion and arithmetic reasoning) were found by Schneck to be the best measures of his V and N (24, p. 30). The reliability coefficients of the memory battery ranged from .637 to .863. Except for analogies, the reliability of which was .702, the verbal and number tests had high reliabilities,—from .891 to .932.

(3) *Results.* Examination of average intercorrelations among her eight tests, as well as a tetrad analysis of the battery, convinced Anastasi of the reality of the memory factor previously identified, and of its independence of V and N (3, pp. 13-25).

This second study of memory organization in an adult group was selected for factor analysis because, in addition to the memory tests, four tests (two of verbal and two of number) measuring presumably independent abilities were included in the battery. These additional reference tests made it possible to check the independence of the hypothetical memory factor more efficiently than could be done with the battery of Table 7. The weights of each of the eight tests in the first four centroid factors, and their communalities, are shown in Table 9. The first three factors account for 28%, 16% and 11%, respectively, of the total variance. The fourth centroid factor which accounts for 3% of the total variance is virtually zero in six of the eight tests, being practically specific to tests 4 and 8, in which it has the same sign. The first three factors account for 55% of the total variance.

Examination of the signs of the factor loadings in II shows a clear differentiation between the verbal-number tests on the one hand and the memory tests on the other. In III, the factor loadings in the verbal and number tests are opposed in sign. These facts and the smallness of IV seemed to justify a description of the intercorrelations of our eight tests in terms of three primary abilities. In Figure 3, the augmented factor weights in II have been plotted against the augmented factor weights in III, around I as a pivot. Inspection of the diagram shows three fairly distinct

TABLE 9

FACTOR LOADINGS IN FOUR CENTROID FACTORS, AND COMMUNALITIES, OF THE EIGHT TESTS IN ANASTASI'S SECOND STUDY

Tests	I	II	III	IV	I^2	II^2	III^2	IV^2	h^2
1. Vocabulary	.432	—.450	—.411	.198	.187	.203	.169	.039	.598
2. Analogies	.562	—.459	—.488	—.095	.316	.211	.238	.009	.774
3. Number Series Completion	.488	—.300	.505	—.089	.238	.090	.255	.008	.591
4. Arithmetic Reasoning	.577	—.333	.389	—.257	.333	.111	.151	.066	.661
5. Word-word	.587	.523	—.115	.026	.345	.274	.013	.001	.633
6. Picture-number	.510	.403	.046	.148	.260	.162	.002	.022	.446
7. Retained Members	.573	.484	.190	.063	.328	.234	.036	.004	.602
8. Syllable Recognition	.449	.143	.164	—.296	.202	.020	.027	.088	.337
Σk^2					2.209	1.305	.891	.237	4.642
$\frac{\Sigma k^2}{N}$					.276	.163	.111	.030	.580

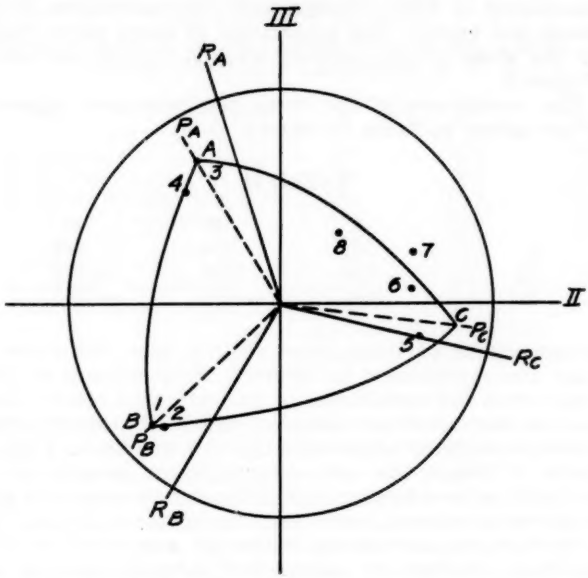

Figure 3

Plot of the Factor Loadings of the Eight Tests of Table 9, with reference to the Centroid Axes II and III.

clusters of tests:—tests 3 and 4 in quadrant 2, tests 1 and 2 in quadrant 3, and tests 5, 6, 7, and 8 in quadrants 1 and 4. If three great circles are drawn upon the surface of the sphere, of which our diagram is a cross-section, each test vector will lie in or close to one of the three planes determined by these circles.

The coordinates of the three primary ability vectors, P_a, P_b, P_c, are given in Table 10:

TABLE 10

	P_a	P_b	P_c
I	.639	.592	.578
II	—.393	.800	—.602
III	.662	.100	—.550

Each column of Table 10 represents the coordinates of one primary test vector. The projections of these three vectors upon the plane of the centroid axes II and III are shown in Figure 3.

The coordinates of the three new reference vectors or axes are given in Table 11 below:

TABLE 11

	A	B	C
I	.502	.634	.490
II	—.269	.756	—.456
III	.821	—.163	—.743

The projections of these three vectors upon the plane II-III are also represented in Figure 3. Each column of Table 11 represents the coordinates of one reference vector. Table 11 is also the matrix by means of which the factor weights in Table 9 are transformed into the new weights in Table 12.

Table 12 shows the new rotated factor weights of our eight tests with reference to the three new reference axes. The entries in column I′ are taken to represent number ability,—defined by arithmetic reasoning and number series completion. Column II carries the memory loadings and column III the verbal. Figure 3 and Table 12 clearly reveal simple structure in the battery. Moreover, the structure is probably unique although no tests lie along the arcs AB and BC. With the present limited battery, the identification of three primary abilities is quite satisfactory.

The rotation effected by the matrix of Table 11 is an oblique, and not an orthogonal, transformation. Hence, our three primary abilities are correlated. The cross products of the entries in Table 10 give the cosines of the angles between the trait vectors, P_a, P_b, P_c, and from these cosines we know at once the correlations between our postulated abilities. Calculation reveals a correlation between P_a and P_c (number ability and memory) of .00:—the reference axes P_a and P_c are orthogonal. The correlation between P_a and P_b (verbal and number abilities) is .242—a rather striking confirmation of Schneck's result (p. 265). The correlation between P_b and P_c (verbal ability and memory) is —.085: again, axes P_b and P_c are virtually orthogonal.

With respect to memory, our results verify Anastasi's finding that in adult groups memory (as defined by her tests) is a primary ability independent of verbal and number abil-

TABLE 12

FACTOR WEIGHTS OF ANASTASI'S EIGHT TESTS IN THREE PRIMARY ABILITIES A, B, C

Tests	I′	II′	III′
1. Vocabulary	.001	.001	.722
2. Analogies	.004	.089	.847
3. Number Series Completion	.741	.000	.001
4. Arithmetic Reasoning	.699	.051	.146
5. Word-word	.060	.786	.135
6. Picture-number	.186	.621	.032
7. Retained members	.314	.698	—.081
8. Syllable Recognition	.322	.356	.033
	(Number)	(Memory)	(Verbal)

ities. Further, our factor analysis shows that tests 7 and 8 (retained members and syllable recognition) are more closely related to number ability than to verbal ability. This last result is probably not as anomalous as it seems. Both tests involve symbol manipulation. Hence, if our number ability (as seems probable) is concerned with manipulation of abstract quantities or with "reasoning" rather than with number *per se*, we should expect a high correlation between tests 7 and 8 and our number factor.

3. Bryan's Study (6).

(1) *Subjects.* The data used in the present analysis were obtained from one hundred boys. The age range was from 5 to 6 years, the mean age being 5 years 6 months.

(2) *Tests.* Eleven tests of memory were administered. These tests made use of a wide variety of materials:—pictures, objects, forms, colors, blocks. In order not to penalize the timid child, verbal responses were kept at a minimum. The tests and their administration are described in detail by Bryan (6, pp. 14-25). In addition to the memory battery, each child was given a vocabulary test; also his Stanford-Binet MA was determined. The reliabilities of most of the memory tests were low,—only three having reliability coefficients above .80. This low reliability was perhaps inevitable, in view of the age of the subjects.

(3) *Results.* From an examination of average intercorrelations, and of the mean tetrads yielded by her memory battery, Bryan concluded that a general factor of memory was present in her tests. An interesting outcome was the finding

that the memory tests were in general as closely related to the Stanford-Binet MA as they were *inter se*. This fact suggests that retentivity is an exceedingly good indicator of general intelligence in young children.

Table 13 presents a factor analysis (centroid method) of the intercorrelations of Bryan's memory tests plus the vocabulary test. Because of the low individual test reliabilities, all of the *r's* were first corrected for attenuation. Test 11 (Faces and Names, delayed) was omitted from the battery because it is simply test 1 repeated after an hour. To correct the correlation between tests 1 and 11 by dividing their obtained correlation by the geometric mean of their intercorrelations boosts the corrected r to 1.00,—a far too optimistic figure. Stanford-Binet MA was omitted because this test is probably a sampling of many psychological functions. It would be idle to expect factor analysis to deal with scores of this sort unless the battery were greatly extended in scope and in variety. Even then the value of one's results would be doubtful (30).

The analysis of Table 13 was not carried beyond four factors since (1) no definite structure was revealed by the first four factors and it seemed imprcbable that additional factors would yield further information; (2) a factor analysis of the uncorrected *r's* yielded a fourth factor which was extremely small.

Investigation of the factor loadings in Table 13 reveals even greater difficulties, so far as interpretation is concerned, than were encountered in Table 7. Except for the vocabulary test, all of the tests in the battery are concerned with recall or recognition,—i.e., with retentivity. Further, the variety of the materials and their appeals to many interests, undoubtedly produced marked variations in attention, cooperation, and effort among these five year old subjects. The lack of reference tests and the opportunity afforded for wide variations in performance would serve to render difficult the isolation of a memory factor, assuming such to be present. While fully recognizing these difficulties, we have, nevertheless, attempted to rotate the test configuration of Table 13 into a meaningful psychological structure. The results of a series of orthogonal transformations are shown in Table 14.

The order of rotation was as follows: first, factor weights in I were plotted against corresponding factor weights in II. All of the memory tests in the diagram fell in quadrant 1, except tests 8 and 9 which, together with the vocabulary

TABLE 13

FACTOR WEIGHTS IN FIRST FOUR CENTROID FACTORS, AND COMMUNALITIES, FOR BRYAN'S ELEVEN TESTS (CORRECTED CORRELATIONS)

Tests	I	II	III	IV	I^2	II^2	III^3	IV^2	h^2
1. Faces and Names	.622	—.649	.264	—.179	.387	.421	.070	.032	.910
2. Picture Recognition ...	.768	.455	.330	.159	.590	.207	.109	.025	.931
3. Objects	.705	.136	.282	.178	.497	.018	.080	.032	.627
4. Colored Forms	.870	.230	.063	.271	.757	.053	.004	.073	.887
5. Paired Pictures	.751	.243	—.350	.223	.564	.059	.123	.050	.796
6. Cube Imitation	.589	.119	—.304	—.340	.347	.014	.092	.116	.569
7. Figured Forms	.808	.235	.138	—.247	.653	.055	.019	.061	.788
8. Digit-Span	.573	—.242	—.154	—.104	.328	.059	.024	.011	.422
9. Sentence-Span	.713	—.189	—.368	.210	.508	.036	.135	.044	.723
10. Colored Designs	.666	.386	—.067	—.187	.444	.149	.004	.035	.632
12. Vocabulary	.763	—.153	—.126	.268	.582	.023	.016	.072	.693
Σk^2					6.299	1.323	.859	.571	9.052
$\frac{\Sigma k^2}{N}$					.525	.110	.072	.048	.754

test, fell in quadrant 4. Test 1 which proved to be decidedly a maverick also fell in quadrant 4. A counter clockwise rotation of 40° about axes III and IV brought all of the test vectors into the positive area. The matrix of the orthogonal transformation was

$$\begin{pmatrix} .766 & -.643 \\ .643 & .766 \end{pmatrix}^{*}$$

Neither rotated I or II seemed to represent any clear-cut, identifiable ability. Hence, the factor weights in rotated I were plotted against those of III and another orthogonal transformation carried out. A rotation of 35° (counter clockwise) made all of the test loadings positive. The matrix of transformation was

$$\begin{pmatrix} .819 & -.574 \\ .574 & .819 \end{pmatrix}^{**}$$

The new first factor loadings of our eleven tests appear in Table 14 under column I′. This trait vector seems to bear the memory loadings,—and we have called it the "memory factor." The rotated factor weights in III appear in Table 14 under II′. This factor seems to represent ability to follow sequences, and probably involves, too, attention and interest.

The last two factors represented by columns III′ and IV′ were obtained by rotating the factor loadings in the already rotated II against IV. The new factor weights are shown in Table 14 under III′ and IV′. The first vector (III′) is apparently concerned with verbal expression and word knowledge; the second (IV′) with interest in manipulation, plus, perhaps, ability with non-verbal material.

While the configuration in Table 14 does not fulfil the conditions for simple structure (p. 278), the psychological meaning of the loadings in the table is, for the most part, reasonable. All of the tests have large weights in the memory column, except (1) paired pictures which is most heavily represented in column two (sequences); (2) cube imitation, which is most heavily weighted in II′ and in IV′ (non-verbal, manipulation); and (3) digit-span and sentence span which cut across all of the factors. Bryan (6, p. 37) found that tests 2, 4, and 7 (picture recognition, colored forms, and

* Trait vector II′ was reflected in order to change the signs of its factor loadings.

** Trait vector III′ was reflected in order to change the signs of its factor loadings.

TABLE 14

NEW FACTOR WEIGHTS OF BRYAN'S ELEVEN TESTS (TABLE 13) AFTER A SERIES OF ORTHOGONAL TRANSFORMATIONS

Tests	I′	II′	III′	IV′
1. Faces and Names	.200	—.182	.560	.723
2. Picture Recognition ...	.911	.236	.213	—.025
3. Objects	.676	.129	.380	.095
4. Colored Forms	.703	.415	.467	.046
5. Paired Pictures	.398	.707	.370	.027
6. Cube Imitation	.258	.552	—.006	.446
7. Figured Forms	.710	.329	.095	.409
8. Digit-Span	.144	.288	.350	.442
9. Sentence-Span	.136	.544	.593	.237
10. Colored Designs	.583	.490	—.023	.228
12. Vocabulary	.326	.382	.635	.197
	(memory)	(sequences)	(verbal expression?)	(manipulation?)

figured forms) were most highly related to her memory battery as a whole. The average correlation of these tests with the whole battery was .430. These three tests also have heaviest loadings in test vector I′. The vocabulary test has a fair weight in "memory," but is most heavily represented in III′ (language-expression, word knowledge). Vocabulary also enters into sequences (II′) and into manipulation (IV′). An r must be greater than $\pm.201(\pm 3PE)$ to be probably significant, so that it is doubtful whether any of the negative entries in Table 14 implies true inverse relationship.

From the point of view of clear-cut structure, the factor analysis of Table 14 is the least satisfactory of any we have made. The homogeneity of the battery; the variability of performance due to the youth of the subjects; the great variety of materials with their differential appeal to interests and attention; timidity and emotional disturbances:—all of these influences, undoubtedly, contributed to our inability to extract clear-cut factors. The analysis in Table 14 was finally decided upon after many trials in which oblique as well as orthogonal transformations were attempted.

Table 14 probably substantiates Bryan's claim for a general memory factor through her tests; and at the same time shows clearly the importance of language (vocabulary) in the activities of young children. Thus, vocabulary has a positive weight in all four factors. In contrast to Bryan's results in which the correlation between vocabulary and the

memory battery was .446, Anastasi (p. 280) found a correlation between the same two variables of .058 (adult subjects). Schneck's and Schiller's data (p. 273) suggested the greater homogeneity of verbal and number abilities in young children than in adults. And Bryan's data in contrast to Anastasi's (p. 285) indicate the same lack of differentiation among the verbal and memory abilities of young children.

The factor analyses of the test batteries in Tables 7, 9 and 13 have supplied insight into the relationships of certain abilities; and have also furnished further evidence (10) of the difference with respect to organization between the mind of the child and the mind of the adult. This type of analysis is a useful supplement to other analytic methods, since it not only enables the investigator to isolate more than one trait within a battery, but also to investigate the strength of trait relationships.

III. EVALUATION OF RESULTS

1. *Applicability of Factor Analysis to the Studies here Considered*

There are two methods of attacking the problem of trait organization. The first is to administer as large and as representative a battery of tests as possible; analyze the intercorrelations into independent (if arbitrary) components; study the factor loadings and signs for clues as to ability clusters; and, finally, perform stimultaneous or successive rotations of the arbitrary axes in order to bring out structure (if present) among the test vectors, and give psychological meaning to the new or rotated factors. This method is advocated by Thurstone (28, Chaps. 1 and 2) (29), and Kelley (19); it is certainly the best—and perhaps the only—method of discovering the size and extent of common connecting bonds over a wide area of measurable abilities. The second method of studying trait organization is not directly concerned with the simultaneous location of many traits, but rather with the identification and measurement of some single hypothetical trait. In this scheme, a large number of tests are gathered—all designed to measure the trait under investigation. Intercorrelations are then analyzed for evidence of a single common ability. If such a trait is discovered, the correlations of this central factor with each test are computed. As a final step, the scores in this central ability may be estimated by means of a regression equation, and these estimated scores validated

against other criteria. This is the method originally employed in the studies which have been considered in this paper. It has been noted in several places that none of the batteries used in these studies covers a very wide range of aptitudes. For this reason, our data are not particularly well adapted to factor methods. In spite of recognized limitations, however, factor analysis has served (1) to substantiate in a more clear-cut fashion the reality of the traits which had been previously reported; and (2) to give, in addition, new information with respect to minor factors not hitherto identified. It should perhaps be pointed out that this second method is really supplementary to the first. Having identified and measured a trait, we must always study it in its relationships to other traits before evaluating its importance in achievement.

2. *Methods of Factor Analysis*

In the analyses reported in this paper, we have consistently applied Thurstone's centroid method of factor analysis (28, Chap. 3). All things considered, this method is perhaps the most satisfactory developed so far. It is easier to apply than the methods of principal axes (28, Chap. 4), or principal components (18) or their variants (20); and just as satisfactory as far as interpretation of results is concerned. The main objections which have been offered to the centroid method are (1) that the first factor extracted by it is a meaningless *pot pourri* (20, p. 61); and (2) that the placing of communalities in the main diagonal of the correlational matrix instead of reliability coefficients or 1.00's (when *r's* have been corrected for attenuation) substitutes an instable for a stable statistic. In reply to the first objection, one may point out, first, that weights in the first factor are based upon intertest correlations, and hence are necessarily average values, whatever the method employed; secondly, that average values are useful in enabling an investigator to "spot" those tests which are most representative of the battery as a whole; and finally, that the first centroid factor is essentially a first approximation to the first principal component (12), and usually differs negligibly from it (28, pp. 131-132). The first—and in fact all of the subsequently calculated factors—are psychologically meaningless until subjected to orthogonal or oblique transformations. Rotation of reference axes (trait vectors) breaks up the "hodge-podgeness" of the first centroid factor (see p.

267) and brings to light structure (if present) which may have been obscured by the averaging technique.

It is true that ordinarily the reliability coefficient is a more stable value than the estimate of communality of each test (largest r in the column) placed in the main diagonal of the correlational matrix. But the charge that it is an "inefficient statistic" may be brought against the reliability coefficient as well as against the estimated communality. Both coefficients are estimates of "true" values, and both are subject to error. To be sure, the reliability coefficient will not change in value as the test is shifted from one battery to another, unless N changes; while our estimate of communality will often change markedly under these conditions. Discrepancies as between estimated communality values and h^2 entries may, however, be reduced to the point of non-significance by making a second and even a third factor analysis of the same data, using each time h^2 values calculated in the previous analysis (for illus., see 14, p. 496). This procedure while useful is not always necessary. In Table 3, the average difference between h^2 entries and estimated communalities used in calculating the first factor was .033, in Table 7 .050, and in Table 9 .044. Ordinarily, unless discrepancies are larger than these a repetition of the factor analysis is not called for.

There are several cogent reasons why communalities instead of reliability coefficients should be placed in the main diagonal of the correlation matrix. Thurstone has stated the case for communalities as follows (28, pp. 77-91): since reliability coefficients will usually contain factors specific to the given tests as well as factors common to the battery, in order to reproduce the obtained correlations *some* of the factors extracted from the correlation matrix must necessarily be specifics, when reliability coefficients are placed in the main diagonal. Stated geometrically, some at least of the trait vectors must show significant projections from only one test vector when reliability coefficients are used. In order to obtain a factor matrix which includes *only* common factors, then, communalities (which exclude specifics) must be used in the main diagonal and not reliability coefficients. The objection to reliability coefficients applies as well to 1.00's, which are placed in the main diagonal of a correlation matrix in which r's have been corrected for attenuation.

3. *The Value of Factor Analysis as a Psychometric Method*

Far more important than arguments concerning details of technique are the objections raised against the validity of factor analysis as a truly psychological method. It will be profitable to consider, first, some of the more persistent of these criticisms, and with this background in mind attempt an evaluation of the role of factor analysis in psychology.

(1) An often recurring objection is made that factors extracted from a table of correlations are numerical *artifacts*: —examples of good algebra, perhaps, but only decimals after all containing little of psychological value. There are many situations in which this contention is justified (9). Since factors are derived from correlation coefficients, they represent a reshuffling of the given numerical data into new categories in accordance with certain rules. A factor matrix is the "best" factor pattern out of many possible solutions; but the best pattern in a mathematical, not a psychological, sense. Unless the reference axes can be so rotated that the test vectors display a meaningful configuration, factor loadings obtained by *any* method are indeed decimal fractions of doubtful psychological value. If, however, the test battery can be described in terms of "primary" traits,—if there is unique simple structure (p. 278)—factor weights are psychologically meaningful. A final and conclusive check upon the psychological reality of a primary trait is validation against independent criteria.

(2) Criticism is often directed against the doctrine of "mental elements" implicit in the theory underlying factor methods. Mental elements imply independence, and from independent elements or "factors" it is an easy step to a view of mind or personality in which elementary units of behavior—like pennies or dice—are tossed into different combinations by the "laws of chance." G. W. Allport (1) has been most forceful in criticizing the doctrine of psychological elements as applied to personality.

While stress has been placed upon the independence of factors by many investigators (28, p. 48), (20), (21) perhaps because of the greater ease in dealing with uncorrelated variables, the assumption of strict (i.e., mathematical) independence among factors is not necessary to factor theory. This is probably fortunate, as the large language overlay in mental tests, the similarity in background, in training, and even in modes of thought of most of the subjects tested in factor studies, make it highly improbable that primary

traits which are actually independent will be extracted from correlation tables. The structure of correlated traits exhibited by the data of Schneck, Schiller, and Anastasi (pp. 264-270-276) is much more likely in view of other evidence than is a forced structure of independent traits. The latter structure, in fact, would be difficult to explain. Freed of the requirement of independence, factors need no longer be identified with unitary mental elements. Present factor methods provide for any degree of relationship from zero to identity among traits.

(3) The lack of *uniqueness* in a given factor pattern has often been advanced as a crucial objection to factor methods. That many factor patterns may be calculated from the same data is undeniably true (17). But it is also true that the patterns obtained by use of the centroid method, principal axes, and principal components methods are "better" descriptions than those got by cut-and-fit methods in the sense of being better mathematical "fits" (28, Chap. 2). Having obtained the best mathematical fit, we often find it possible to reveal underlying psychological structure or order among the tests of our battery through axis rotation. Thurstone (28, pp. 155-156) has laid down three conditions for uniqueness of simple structure which are probably necessary and sufficient.* When a factor pattern fulfills these conditions (see pp. 278-279) it is no longer simply "one among many"—but is unique in a psychological as well as in a mathematical sense.

Tryon (31) has objected to the principle of parsimony usually employed by factorists in the isolation of primary traits. He regards as biologically more tenable the view that abilities are determined by a host of independent factors as large in number as the independent genetic elements. Tryon's view is interesting, but not very probable. It seems more likely that in the course of development, learning and the formation of habit hierarchies would tend to bring out a few broad factors (11). The elementary genetic factors may still be there, to be sure, but they are no longer identifiable as such. Practically, some principle of combination, selection, and consolidation must be stipulated if one is to account for the difference among individuals.

(4) The charge that factor analysis is essentially "faculty psychology" with a frontage of mathematical formulas is a specious and not very exact characterization. Factor analysis

* Godfrey Thomson has investigated the requirements under which the second condition will hold. See Psychometrika, 1936, 1, 155-163.

does indeed attempt to discover fundamental traits which may be thought of as conditioning or governing performance in a variety of tasks. But factors differ from faculties in being (1) experimentally determined, (2) operationally defined, and (3) subject to tests of validity. Faculties begin typically as *a priori* generalizations and end as reified forces. Factors begin and end as descriptions of common traits which cut across the performances in which they are found to be present.

(5) The *names* given to factors are often a source of suspicion. When factors are derived from mental tests involving a wide range of abilities, a hypothetical primary trait is defined strictly in terms of the test cluster in which it appears with greatest weight. This procedure gives reasonable results when the test battery contains a variety of tests differing sharply in material dealt with and in operations demanded (p. 284). When (1) a test score is perhaps a congeries of many traits (e.g., I.Q.), or when (2) a battery contains tests of only one sort (e.g., p. 289) naming of factors becomes almost an impossibility. This difficulty is greatly enhanced when personality tests or questionnaires are analyzed factorially. Factor analysis has been used with success in the isolation of fairly objective personality traits (8) (15). But the complexity of the data obtained from rating scales and questionnaires; the many different determinants of the same trait in different individuals; the difficulty in finding a definitive label for a factor:—all of these conditions imply that factor analysis is not especially well adapted to the study of personality. This does not mean that factor analysis may not be of decided value in limiting the field of inquiry, breaking down nebulous concepts, and redefining proposed traits (1).

4. *Summary*

Factor analysis has proved useful (1) as a technique in test construction (4, 5), (2) in investigating the organization of abilities, and (3) as a first step in defining and measuring common personality traits. Factor analysis has already yielded valuable information concerning mental abilities. Verbal ability, numerical ability, mental speed (7), memory and perhaps other (29) primary traits have been isolated and their validity fairly well established. In the isolation of certain common personality traits, also, factor analysis has been valuable (15). Freed of the need for assuming independence among traits, and given really valid

data, factor analysis should find increased usefulness as a psychological method.

In drawing conclusions with respect to age, sex, race and other variables, students of differential psychology have always been handicapped because their conclusions rest so largely upon results from tests of unidentifiable functions. The scores on many test batteries (e.g., those of general intelligence) are conglomerations of so many abilities that it is difficult to know exactly what an obtained difference signifies. Differences among groups, expressed in terms of primary traits, will be far more enlightening than differences expressed in terms of tests the meaning of which is not clear. A knowledge of primary traits may lead, too, to a revision of many present beliefs. Studies in the transfer of training, in the effects of practice under various conditions, in the growth and decline of mental abilities, may be mentioned as promising fields of work. Experimental investigation here is greatly needed.

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